PHYS 320 ANALYTICAL MECHANICS

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Fall 2108

TODAY

Newton's Laws

Review of statics

Statics: next steps … trusses

Static Equilibrium

- Conditions of equilibrium:

$$\vec{F}_{net} = \sum_i \vec{F}_i = 0$$

2nd Law

$$\vec{\Gamma}_{net} = \sum_i \vec{\Gamma}_i = 0$$

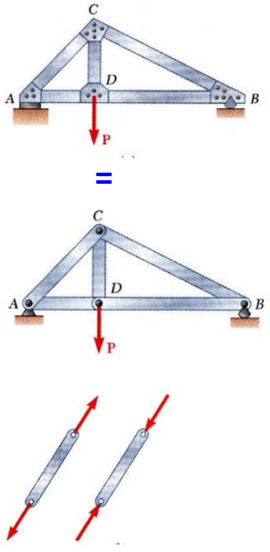
$$\vec{\tau}_{net} = \sum_i \vec{\tau}_i = 0$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

3rd Law

Static Equilibrium: Structures

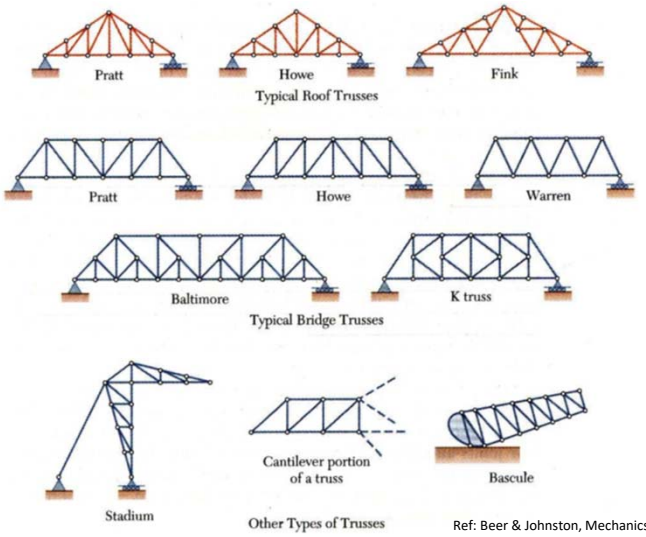
- **Trusses**:
  - A truss consists of straight, two-force **members** connected at joints; no member is continuous through a joint.
  - Most structures are made of several trusses joined together to form a 3D framework. Each truss carries those loads which act in its plane and may be treated as a 2D structure.
  - Bolted or welded connections are modeled as pinned together; forces acting at member ends reduce to a single force.
  - If forces tend to pull the member apart, it is in **tension**. If the forces tend to compress the member, it is in **compression**.
  - Truss members assumed to have negligible mass (compared to the loads they carry).



Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures

- Some sample **trusses**:



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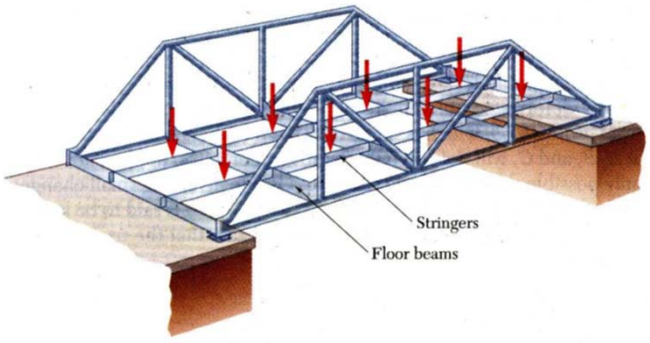
Static Equilibrium: Structures

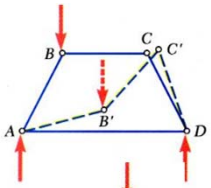


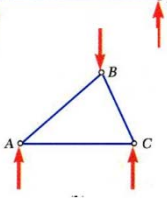
Truss members are slender & incapable of supporting large lateral loads

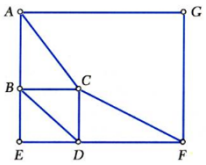
- Loads must be applied at the joints.

Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures







- A *rigid truss* will not collapse under the application of a load
- A simple truss constructed by successively adding two members and one joint to basic triangular truss.
- Simple truss: $m = 2n - 3$

  where m = total number of members

  n = number of joints

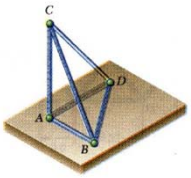
Ref: Beer & Johnston, Mechanics for Engineers: Statics

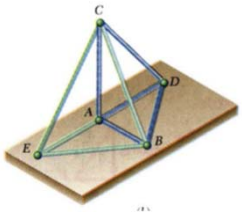
Static Equilibrium: Structures

- *Elementary space truss*:

  6 members connected at 4 joints to form a tetrahedron
- *Simple space truss:*

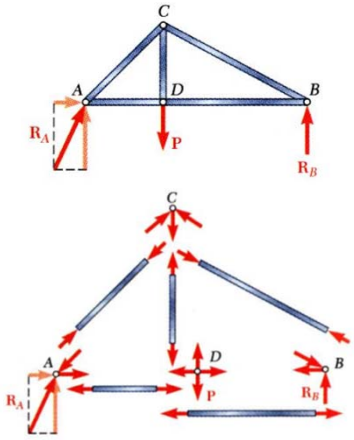
  formed and extended when a unit of 3 new members + 1 joint are added; $m = 3n - 6$
- Equilibrium conditions of joints: $3n$ equations
  - ➢ can determine m member forces & support reactions





Static Equilibrium: Structures

- Analysis of Trusses: the Method of Joints

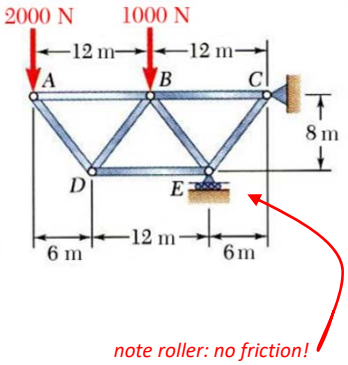


m = # members = 5
n = # joints = 4

- Dismember the truss. Create FB diagram for @ **member** & joint.
- Two forces exerted on any **member** are equal, have the same *line of action* but opposite direction.
- Forces exerted by a **member** on the joints at its ends are 3rd law pairs with forces on the **member** (equal and opposite)
- Conditions of equilibrium on the joints provide *2n* equations for *2n* unknowns. For a simple truss, *2n = m + 3*. Solve for *m* **member** forces and *3* reaction forces at the supports.
- Conditions for equilibrium for the *entire truss* provide *3* additional equations which are not independent of the pin equations (check!).

Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example



Find the force in each member of the truss!

Plan of attack:

- F-B diagram of entire truss; find reactions at supports
- Joint *A* has only two unknown member forces; find these first
- Next analyze joints D, B, & E.
- Use equilibrium conditions to check results at C.

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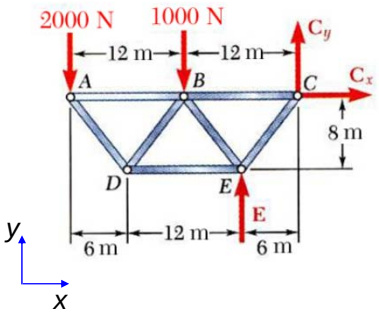
Static Equilibrium: Structures Example



$$\sum_{\text{about } C} \vec{\Gamma} = 0$$

$$\Rightarrow \quad \vec{E} = 10{,}000N\ \hat{j}$$

$$\sum F_x = 0$$

$$\Rightarrow \quad C_x = 0$$

$$\sum F_y = 0$$

$$\Rightarrow \quad C_y = -7{,}000N$$

(we chose the wrong direction in diagram!)

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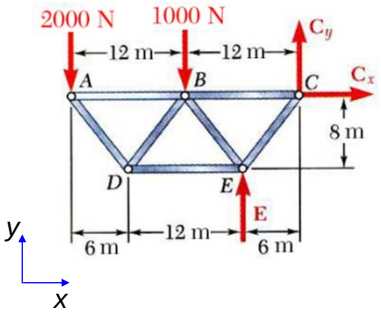
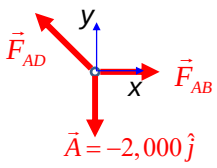
Static Equilibrium: Structures Example



Joint A:



$$\sum F_x = 0 = F_{AB} - F_{ADx}$$

$$\sum F_y = 0 = F_{ADy} - A$$

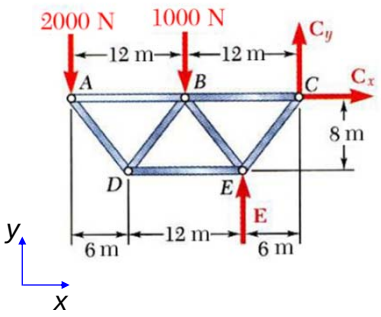
$$\Rightarrow \quad F_{AD} = 2{,}500N$$

$$F_{AB} = 1{,}500N$$

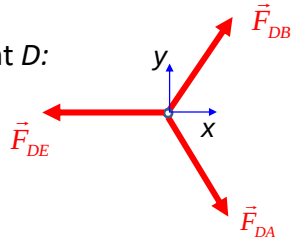
Member AD is in compression.
Member AB is in tension.

Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example



Joint *D:*



$$\sum F_x = 0 = F_{DBx} + F_{DAx} - F_{DE}$$

$$\sum F_y = 0 = F_{DBy} - F_{DAy}$$

$$F_{DB} = F_{DA} = 2,500N$$

$$F_{DE} = 3,000N$$

Member DB is in tension.
Member DE is in compression.

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